Kuwait University | Math 101 | January 9, 1995
Math. Department | Final Exam | Duration : 2Hrs.

Answer the following questions. Q_1, Q_2, Q_3, Q_4, Q_5, 6 Marks each, Q_6 8 Marks and Q_7 12 Marks. **Calculators are not allowed.**

1) Evaluate each of the following limit if it exists:-

a) $\lim_{x\to 8} \dfrac{3x^{\frac{2}{3}} - 12}{x^{\frac{1}{3}} - 2}$ b) $\lim_{x\to 0} \dfrac{1-\cos(2x)}{x\tan 3x}$

2 a) Let $f(x) = 1 - x^3 - \sin 2x$. Show that the equation $f(x) = 0$ has a unique real root in the interval $[0, \frac{\pi}{4}]$

b) Evaluate the integral $\int_{-3}^{3} \sqrt{9-x^2}\, dx$.

3) a) Use the Mean Value Theorem to prove that if $f'(x) = 0$ on an interval I then $f(x)$ is constant.

b) Show that the function $f(x) = \int_0^{\frac{1}{x}} \dfrac{dt}{(t^2+1)} + \int_0^{x} \dfrac{dt}{(t^2+1)}$

is constant for x> 0.

4) A man wants to design a rectangular garden with a fence around it. The fencing for three sides of the garden costs 2KD per meter, and the fencing for the fourth side costs 3KD per meter. If the man has 120 KD to spend on the fence, find the dimensions of the garden that maximize its area.

5) Let $f(x) = \begin{cases} 3\sqrt{x} & if \quad 0 \le x < 4 \\ 2x - 2 & if \quad 4 \le x < 6 \end{cases}$

a) Find the average value of $f(x)$ on [0,6].

b) Find the number $z \in (0,6)$ that satisfies the conclusion of the MeanValue Theorem for definite integrals.

6) Evaluate:- a) $\int \dfrac{x^2-2}{(x^3-6x+1)^6} dx$ (b) $\int_0^1 (x^2-1)^{10} x^3\, dx$

7) Let $f(x) = \dfrac{1}{x^2-1} + 2$

a) Find the intervals on which f is increasing or decreasing, and the local extrema, if any.

b) Find the intervals on which f is concave upward or downward, and the points of inflection , if any.

c) Find the vertical and horizontal asymptotes for the graph of f , if any.

d) Sketch the graph of the function.